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Soft-gluon resummation in quark-vector boson vertex at high energy

On-mass-shell case

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Abstract. Resummation of the soft-gluon radiative corrections for the quark-vector boson vertex is performed within the path-integral (world-line) approach. The leading-order expression for the vacuum-averaged Wilson integral for an arbitrary gauge field is found in n-dimensional space-time. The cusp anomalous dimension of the color non-singlet Sudakov form factor of *on*-mass-shell quark is calculated in an arbitrary covariant gauge in the one-loop order, and the leading double-logarithmic asymptotical behavior is obtained from the corresponding evolution equation.

PACS. 12.38.Cy Summation of perturbation theory – 12.38.Lg Other nonperturbative calculations

1 Introduction

Resummation of the logarithmic corrections in high-energy processes is required to justify effectiveness of the Standard Model at highest energies accessible at modern and future colliders (see, *e.g.*, [1] and references therein). In electroweak processes, the Sudakov corrections can influence profoundly the cross-sections at e^+e^- linear colliders at TeV energies, and the precise evaluation of them is quite important in search for the New Physics, as well as to test the predictive power of the Standard Model [2]. In the strong interactions sector, the form factors of quarks are the most elementary entities exhibiting the double-logarithmic behavior. The quark form factors being, of course, unobservable quantities, enter into the quark-photon and quark-gluon vertices in the calculations of various QCD processes at the partonic level, and are of a special theoretical as well as phenomenological importance [3,4]. Very recently, the investigations of the form factors of quarks has been connected with the progress in experimental study of the constituent quarks and search for the perturbative and non-perturbative effects in their structure from low to high energy [5–7].

The form factors of the elementary fermions at large momenta transfers in gauge theories —in QED, and later in QCD— had been studied extensively since the fifties [8–12]. In the color singlet case —corresponding to the elastic *on*-shell quark scattering in an external electromagnetic field— the exponentiation of the infrared singularities has been proved and the correct leading asymptotic at high energy has been obtained [10,13–15]: the high-energy behavior of the on-mass-shell quark form factor can be described in terms of the perturbative evolution equation

$$F^{qq\gamma}\left[Q^2\right]/F^{qq\gamma}\left[Q_0^2\right] = \exp\left(-\int_{Q_0^2}^{Q^2}\frac{\mathrm{d}\mu}{2\mu}\ln\frac{Q^2}{\mu}\Gamma_{\rm cusp}\left[\alpha_s(\mu)\right] + NLO\right). \quad (1)$$

In this case, the so-called cusp anomalous dimension reads in one-loop order [14]

$$\Gamma^{qq\gamma}_{\rm cusp}(\alpha_s) = \frac{\alpha_s}{\pi}C_F + O(\alpha_s^2) > 0, \quad (2)$$

and, therefore, the IR-sensitive part of the electromagnetic quark form factor experiences the Sudakov suppression at large Q^2. On the other hand, it has been shown recently, that in similar electroweak reactions the cusp anomalous dimension has the opposite sign for charged bosons $W^\pm$, and then the enhancement takes place instead of the suppression [16].

Here we apply the powerful Wilson integrals techniques [14,15,17,18] to study of the Sudakov resummation of soft-gluon radiative corrections to the quark-vector boson vertex with large transferred momentum. In particular, we concentrate on the qqg-vertex, since $qq\gamma$-vertex can be trivially restored from the latter. The world-line–based

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formulations of quantum theories are actively developed not only due to the wide range of applications to the standard perturbative QFT, but also from the point of view of various string theories (for recent review, see [19] and references therein).

The very important feature of this approach is that it does not refer directly to the standard perturbative techniques and this allows one to avoid diagrammatic calculations which used to be very difficult and involved in non-Abelian gauge theories. Therefore, it is equally suitable for the perturbative as well as non-perturbative calculations [20]. The non-perturbative calculations within the world-lines approach could be compared with the results obtained in lattice QCD and other non-perturbative frameworks. The quark-vector boson vertex, being one of the fundamental objects in the theory of strong interactions, is under active investigation nowadays. The resummation of the gluon radiative corrections to the qqg-vertex has a direct relation to the problem of IR behavior, chiral symmetry breaking, and confinement in QCD (see, ref. [21] and references therein). In high-energy scattering, it can contribute, *e.g.*, to the amplitude with two-gluon exchange in the t-channel in the color singlet state.

The RG equation and corresponding gauge-dependent cusp anomalous dimension which defines the IR properties of the non-singlet quark form factor were studied for the first time in ref. [12] in the leading logarithmic approximation within the standard diagrammatic approach. In the present paper, we make an attempt to generalize the path-integral formulation to colored quark-vector boson vertices and found in one-loop order the cusp anomalous dimension of the averaged path-ordered Wilson exponential corresponding to the color non-singlet quark form factor. We point out an erroneous sign of one of the terms of the anomalous dimension calculated in [12], and find and check the correct expression. Thus, by virtue of exponentiation, one concludes that the colored-quark form factor demonstrates the enhancement at large Q^2, in contrast to the colorless case, eq. (1). The origin of this enhancement is in close connection with the structure of the gauge group and has the same nature as the similar behavior in the decays of the charged electroweak bosons into fermions [16].

2 Soft-gluon resummation within the world-line approach

In this section, we generalize the world-line method for evaluation of the QCD amplitudes with resummed radiative gluon corrections developed in [15] to the case of the quark-vector boson vertex. For this purpose, let us consider the color non-singlet form factor of a quark which can be extracted from the amplitude of the quasi-elastic (color changing) quark scattering (in the given kinematics) in an external colored (gluon) gauge field: the quark on a mass shell comes from infinity, emits a hard gluon at the origin, changes the color, and goes away to infinity:

$$u_i(p_1)\left[\mathcal{M}_\mu^{qqV}\right]^a_{ij} v_j(p_2) = F^{qqV}\left[(p_1-p_2)^2;\xi\right]\bar{u}_i(p_1)T^a_{ij}\gamma_\mu v_j(p_2), \tag{3}$$

where $p_{1,2}$ are the momenta of the incoming and outgoing quarks, and $a,(i,j)$ are the gluon and quark color indices, respectively. This vertex is gauge non-invariant, therefore the dependence from the gauge parameter ξ is included. We consider the covariant gauge with the following gauge-fixing term in the Lagrangian:

$$\mathcal{L}_{\rm gf}^{\rm COV} = -\frac{\lambda}{2}\sum_a(\partial A^a)^2, \qquad \xi = 1-\frac{1}{\lambda}, \tag{4}$$

while the other important case—the axial gauge $\mathcal{L}_{\rm gf}^{\rm AX} = -\lambda'/2(n\cdot A)^2$—deserves a special attention. For the choice of kinematics with the large (longitudinal) momentum transfer between quarks (Sudakov regime), the interactions of quarks with external gluon, as well as the (external-)gluon-gluon and gluon-ghost interactions were shown to be IR-safe, thus they can be neglected in the considered case [11]. The kinematics is fixed by the small masses of the quarks and large squared transferred momentum:

$$m^2 = p_1^2 = p_2^2\,, \quad (p_1p_2) = m^2\cosh\chi\,, \quad (p_1p_2)\gg m^2, \tag{5}$$

or

$$\begin{aligned}&s=(p_1+p_2)^2 = 2m^2(1+\cosh\chi),\\ &-Q^2 = t = (p_1-p_2)^2 = 2m^2(1-\cosh\chi),\\ &(s+t)\gg(s-t).\end{aligned} \tag{6}$$

According to the Feynman rules, the color structure of the vertex function (3) is determined by the matrix t^a_{ij} in the fundamental representation of the Lie algebra of the color gauge group $SU(N_c)$.

Within the world-line formalism, the two-point (Euclidean) fermionic Green function can be written in terms of the Polyakov path-integral representation [18,15]:

$$\begin{aligned}&G_{ij}(x,y) =\\ &-i\int \mathrm{d}\tau\,\mathrm{e}^{-m^2\tau}\int_{x_0=x}^{x_\tau=y}\mathcal{D}x(\tau')\left[m-\frac{1}{2}\gamma\cdot\dot{x}(\tau)\right]\\ &\cdot\mathcal{P}\exp\left(\frac{i}{4}\int_0^\tau \mathrm{d}\tau'\,\sigma_{\mu\nu}\omega_{\mu\nu}\right)\cdot\exp\left(-\frac{1}{4}\int_0^\tau\mathrm{d}\tau'\,\dot{x}^2(\tau')\right)\\ &\cdot\mathcal{P}\exp\left[ig\int_0^\tau \mathrm{d}x_\rho\hat{A}_\rho(x(\tau'))\right]_{ij}.\end{aligned} \tag{7}$$

Here $\omega_{\mu\nu} = \frac{\tau}{2}(\ddot{x}_\mu\dot{x}_\nu - \dot{x}_\mu\ddot{x}_\nu)$ is the Polyakov spin factor, $\sigma_{\mu\nu} = [\gamma_\mu,\gamma_\nu]$, (i,j) are the quarks color indices, and τ' is treated as the proper time of a fermion with mass m traveling from the point x at $\tau'=0$ to y at $\tau'=\tau$. Now we are interested in the process where a fermion with initial momentum p_1 and color i starts its way at point x, passes through a point z where it experiences both the large momentum transfer $Q=p_1-p_2$, $Q^2<0$ and the color changing described by the matrix T^a_{ij}, and comes finally to point

y with momentum p_2 and color j. This process can be studied in terms of the three-point vertex function

$$V^{\mu}_{ij}(x,y,z) = G_{ii'}(x,z)\Gamma^{\mu}_{i'j'}G_{j'j}(z,y), \quad (8)$$

where the vertex matrix Γ^{μ}_{kl} consists of a Dirac component $\Gamma^{\mu} = \gamma^{\mu}, \gamma_5\gamma^{\mu}\ldots$ and a color component $T^a_{kl} = \delta_{kl}, t^a_{kl}$: $\Gamma^{\mu}_{kl} = \Gamma^{\mu}\otimes T^a_{kl}$. For simplicity, we will consider the last case which corresponds to the quark-gluon vertex, while the color singlet quark-photon result is restored easily. Therefore, in the momentum space, the vertex function reads

$$\begin{aligned}&\left[\tilde{V^{\mu}_{ij}}\right]^a(p_1,p_2) = \\ &\sum_{C^{(1)}_k}\sum_{C^{(2)}_l}\tilde{\Gamma}^{\mu}\left[C_{k,l}\right]\cdot\mathcal{T}\left\{\mathcal{P}\exp\left[ig\int_{C^{(1)}_k}\mathrm{d}x_\mu\,\hat{A}_\mu(x)\right]\right.\\ &\left.\cdot T^a_{ij}\cdot\mathcal{P}\exp\left[ig\int_{C^{(2)}_l}\mathrm{d}x_\mu\,\hat{A}_\mu(x)\right]\right\}, \end{aligned}\quad (9)$$

where the sum over all possible trajectories $C^{(1,2)}_{k,l}$ of two quarks is assumed. The functions $\Gamma^{\mu}[C_{k,l}]$ accumulate information about quark propagators. In this language, the extraction of the soft (long-distance) part can be performed by choosing a special set of paths with simple geometry for ordered exponentials in eq. (9). In our case, one should take for this purpose an angle with semi-infinite sides that represent the classical trajectories of the quarks [14,15]. Then, the UV cutoff which arises in evaluation of the "soft" integrals is identified with the IR cutoff of the "hard" part. After renormalization of the soft exponentials (for details, see refs. [13]), one can find the so-called cusp anomalous dimension which determines the large-Q^2 asymptotic of the form factor

$$\begin{aligned}&F^{qqV}\left[Q^2\right] = \\ &\exp\left(-\int_{Q_0^2}^{Q^2}\frac{\mathrm{d}\mu}{2\mu}\ln\frac{Q^2}{\mu}\Gamma^{qqV}_{\text{cusp}}\left[\alpha_s(\mu)\right] + NLO\right).\end{aligned}\quad (10)$$

By virtue of the eq. (9), it is convenient to express the latter in terms of the vacuum average of the two path-ordered Wilson exponentials:

$$T^a_{ij}\ F^{qqV}\left[Q^2;\xi\right] = \left\langle 0\middle|\mathcal{T}\left\{W_{ii'}T^a_{i'j'}W_{j'j}\right\}\middle|0\right\rangle. \quad (11)$$

Taking into account that eq. (11) contains, in general, both UV and IR divergences, we use scales μ^2 and λ^2 as the UV and IR regulator, respectively. For the on-mass-shell quarks, their trajectories are the semi-infinitely extended paths, and can be parameterized as:

$$\textit{In}:\ x_\mu = v^1_\mu\,\tau, \quad \tau\in[-\infty,0],\ v^1_\mu = p^1_\mu/m, \quad (12)$$

$$\textit{Out}:\ y_\nu = v^2_\nu\,\sigma, \quad \sigma\in[0,+\infty],\ v^2_\nu = p^2_\nu/m. \quad (13)$$

Thus, the path-ordered exponentials can be written as

$$W_{ii'} = \mathcal{P}\exp\left[ig\,t^{\alpha}v^1_\mu\int_{-\infty}^{0}\mathrm{d}\sigma\,A^{\alpha}_\mu(v^1\sigma)\right]\Bigg|_{ii'} \quad (14)$$

and

$$W_{j'j} = \mathcal{P}\exp\left[ig\,t^{\beta}v^2_\mu\int_{0}^{\infty}\mathrm{d}\sigma\,A^{\beta}_\mu(v^2\sigma)\right]\Bigg|_{j'j}. \quad (15)$$

The non-zero contributions (up to $O(g^2)$ terms) to F^{qqg} (11) stem from the terms

$$W_0 = \delta_{ii'}\delta_{j'j}t^a_{i'j'} = t^a_{ij}\,, \quad (16)$$

$$\begin{aligned}W^{(1)}_{LO} =& -\frac{g^2}{2}\,t^a_{ij}\,C_F\,v^1_\mu v^1_{\mu'}\\ &\cdot\int_0^\infty\mathrm{d}\sigma\int_0^\infty\mathrm{d}\sigma'\,\theta(\sigma-\sigma')\,D_{\mu\mu'}\left[v^1(\sigma-\sigma')\right],\end{aligned}\quad (17)$$

$$\begin{aligned}W^{(2)}_{LO} =& -\frac{g^2}{2}\,t^a_{ij}\,C_F\,v^2_\mu v^2_{\mu'}\\ &\cdot\int_{-\infty}^0\mathrm{d}\sigma\int_{-\infty}^0\mathrm{d}\sigma'\,\theta(\sigma-\sigma')\,D_{\mu\mu'}\left[v^2(\sigma-\sigma')\right],\end{aligned}\quad (18)$$

$$\begin{aligned}W^{(12)}_{LO} =& -\frac{g^2}{2}\left(C_F - \frac{C_A}{2}\right)\,t^a_{ij}\,v^1_\mu v^2_\nu\\ &\cdot\int_0^\infty\mathrm{d}\tau\int_0^\infty\mathrm{d}\sigma\,D_{\mu\nu}(v^1\tau+v^2\sigma).\end{aligned}\quad (19)$$

Here the free gluon propagator is

$$\left\langle 0\middle|\mathcal{T}A^{\alpha}_\mu(x)A^{\beta}_\nu(y)\middle|0\right\rangle = \mathcal{D}^{\alpha\beta}_{\mu\nu}(x-y) = \delta^{\alpha\beta}D_{\mu\nu}(x-y), \quad (20)$$

and the following relations have been used:

$$\begin{aligned}t^{\alpha}_{ki}t^{\alpha}_{il} &= \frac{N_c^2-1}{2N_c}\delta_{kl} = C_F\delta_{kl}\,,\\ t^{\alpha}_{ij}t^{\alpha}_{kl} &= \frac{1}{2}\left[\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right].\end{aligned}\quad (21)$$

3 Leading-order contributions for an arbitrary gauge field

First, we evaluate the vacuum averaged Wilson integral eq. (11) in n-dimensional space-time, for an arbitrary gauge field which can be of any origin, for instance— non-perturbative. Then the general results will be applied in the case of usual perturbative gluon field.

It is convenient to present the gauge field propagator $D_{\mu\nu}(z)$ in the form [7]:

$$D_{\mu\nu}(z) = g_{\mu\nu}\partial_\rho\partial^\rho D_1(z^2) - \partial_\mu\partial_\nu D_2(z^2). \quad (22)$$

Calculation of the integrals (17, 19) requires the following expressions:

$$\partial_\rho\partial^\rho = 2n\partial + 4z^2\partial^2, \quad \partial_\mu\partial_\nu = 2g_{\mu\nu}\partial + 4z_\mu z_\nu\partial^2,\ \partial = \partial_{z^2}. \quad (23)$$

The scalar products are:

$$\begin{aligned}\left(v^{1,2}_\mu z_\mu\right)^2 &= (\sigma-\sigma')^2, \qquad v^1_\mu v^2_\nu g_{\mu\nu} = \cosh\chi\,,\\ v^1_\mu v^2_\nu z_\mu z_\nu &= (v^1\sigma+v^2\tau)^2\cosh\chi - \sigma\tau\sinh^2\chi\,.\end{aligned}\quad (24)$$

In order to control UV singularity, the transverse space-like separation $\boldsymbol{b} = (0_\parallel, \boldsymbol{b}_\perp)$ between two points at the integration path is introduced:

$$(\sigma - \sigma')^2 \longrightarrow (\sigma - \sigma')^2 - \boldsymbol{b}^2,$$
$$(v^1\sigma + v^2\tau)^2 \longrightarrow (v^1\sigma + v^2\tau)^2 - \boldsymbol{b}^2. \quad (25)$$

The path-ordered integrals from eqs. (17), (19) can be evaluated using the following basic integrals:

$$\int_0^\infty \mathrm{d}\sigma\, \mathrm{d}\sigma'\, \mathrm{e}^{-\alpha(\sigma-\sigma')^2} = -\frac{1}{2\alpha},$$
$$\int_0^\infty \mathrm{d}\sigma\, \mathrm{d}\sigma'\, \sigma\sigma'\, \mathrm{e}^{-\alpha(\sigma-\sigma')^2} = -\frac{1}{12\alpha^2},$$
$$\int_0^\infty \mathrm{d}\sigma\, \mathrm{d}\tau\, \mathrm{e}^{-\alpha(\sigma^2+\tau^2+2\sigma\tau\cosh\chi)} = \frac{1}{2\alpha}\frac{\chi}{\sinh\chi},$$
$$\int_0^\infty \mathrm{d}\sigma\, \mathrm{d}\tau\, \sigma\tau\, \mathrm{e}^{-\alpha(\sigma^2+\tau^2+2\sigma\tau\cosh\chi)} = \frac{\chi\coth\chi - 1}{4\alpha^2\sinh^2\chi}. \quad (26)$$

Then, applying the Laplace transform to the invariant functions $D_i(u)$, and its derivatives over $u = z^2$:

$$D_i^{\{k\}}(u) = (-)^k \int_0^\infty \mathrm{d}\alpha\, \alpha^k\, \mathrm{e}^{-\alpha u}\bar{D}_i(\alpha), \quad (27)$$

one obtains the following formula:

$$\int_0^\infty \mathrm{d}\sigma\, \mathrm{d}\sigma'\, D'(u) = \frac{1}{2}D(-\boldsymbol{b}^2),$$
$$\int_0^\infty \mathrm{d}\sigma\, \mathrm{d}\sigma'\, D''(u) = \frac{1}{2}D'(-\boldsymbol{b}^2), \quad (28)$$
$$\int_0^\infty \mathrm{d}\sigma\, \mathrm{d}\sigma'\, \sigma\sigma'\, D''(u) = \frac{1}{12}D(-\boldsymbol{b}^2), \quad (29)$$
$$\int_0^\infty \mathrm{d}\sigma\, \mathrm{d}\sigma'\, (\sigma-\sigma')^2\, D''(u) = -\frac{1}{2}D(-\boldsymbol{b}^2) \quad (30)$$

for $u = (\sigma - \sigma')^2 - \boldsymbol{b}^2$, and

$$\int_0^\infty \mathrm{d}\sigma\, \mathrm{d}\tau\, D'(u) = -\frac{\chi}{2\sinh\chi}D(-\boldsymbol{b}^2),$$
$$\int_0^\infty \mathrm{d}\sigma\, \mathrm{d}\tau\, \sigma\tau D''(u) = \frac{\chi\coth\chi - 1}{4\sinh^2\chi}D(-\boldsymbol{b}^2),$$
$$\int_0^\infty \mathrm{d}\sigma\, \mathrm{d}\tau\, (v^1\sigma + v^2\tau)^2 D''(u) = \frac{\chi}{2\sinh\chi}D(-\boldsymbol{b}^2),$$
$$\int_0^\infty \mathrm{d}\sigma\, \mathrm{d}\tau\, D''(u) = \frac{\chi}{2\sinh\chi}D'(-\boldsymbol{b}^2) \quad (31)$$

for $u = (v^1\sigma + v^2\tau)^2 - \boldsymbol{b}^2$.

Thus, one finds the general expressions in n dimensions and arbitrary covariant gauge, with the gauge field two-point correlator expressed like in eq. (22):

$$W^{(1)} = W^{(2)} = -t^a_{ij}\frac{g^2}{2}C_F \cdot \left[(n-2)D_1(-\boldsymbol{b}^2) + 2\boldsymbol{b}^2 D_1'(-\boldsymbol{b}^2) + D_2(-\boldsymbol{b}^2)\right], \quad (32)$$

$$W^{(12)}(\chi) = t^a_{ij}\, g^2 \left(C_F - \frac{C_A}{2}\right) \cdot \left[\chi\coth\chi\left((n-2)D_1(-\boldsymbol{b}^2) + 2\boldsymbol{b}^2 D_1'(-\boldsymbol{b}^2)\right) + D_2(-\boldsymbol{b}^2)\right] \quad (33)$$

up to $O(g^4)$ order terms. Let us emphasize, that eqs. (32), (33) derived above are valid for any gauge field in the adjoint representation of the $SU(N_c)$ color group. Therefore, these results can be used for evaluation of the non-perturbative, *e.g.*, instanton, contributions to the vacuum-averaged Wilson integrals in eq. (11).

4 One-loop perturbative contribution

Now let us apply this general result in the particular case of the perturbative gluon field. Here and in what follows, the dimensional regularization is used with $n = 4 - 2\varepsilon$, $\varepsilon < 0$ in order to regulate the IR-divergent terms in the integrals, respecting, in the same time, the gauge invariance. The gluon propagator in the coordinate space reads

$$D_{\mu\nu}(z;\xi) = \frac{\lambda^{4-n}}{i}\int\frac{\mathrm{d}^n k}{(2\pi)^n}\frac{\mathrm{e}^{-ikz}}{k^2+i0}\left[g_{\mu\nu} - \xi\frac{k_\mu k_\nu}{k^2+i0}\right] = \frac{1}{4\pi^2}\left(-\pi\lambda^2\right)^\varepsilon \cdot \left[g_{\mu\nu}\frac{\Gamma(1-\varepsilon)}{(z^2-i0)^{1-\varepsilon}} + \xi\partial_\mu\partial_\nu\frac{\Gamma(-\varepsilon)}{(z^2-i0)^{-\varepsilon}}\right]. \quad (34)$$

The perturbative one-loop invariant functions in eq. (22) then read

$$D_1(z^2) = \frac{1}{16\pi^2}\frac{\Gamma(1-\varepsilon)}{\varepsilon}\left(-\pi\lambda^2 z^2\right)^\varepsilon,$$
$$D_2(z^2) = -\xi D_1(z^2). \quad (35)$$

Now eqs. (32), (33) contain the IR singularities at $\varepsilon \to 0$. Performing the standard renormalization procedure within the $\overline{MS}$ scheme, described in detail in refs. [13,14,22], one finds

$$W^{(1)}_{LO}(\alpha_s, \mu^2/\lambda^2;\xi) = W^{(2)}_{LO}(\alpha_s, \mu^2/\lambda^2;\xi) = t^a_{ij}\frac{\alpha_s}{4\pi}C_F\left(1 - \frac{\xi}{2}\right)\ln\frac{\mu^2}{\lambda^2}, \quad (36)$$

and

$$W^{(12)}_{LO}(\alpha_s, \chi, \mu^2/\lambda^2;\xi) = -t^a_{ij}\frac{\alpha_s}{2\pi}\left(C_F - \frac{C_A}{2}\right)\left[\chi\coth\chi - \frac{\xi}{2}\right]\ln\frac{\mu^2}{\lambda^2}. \quad (37)$$

Here the UV-normalization point is taken $\mu^2 = 4\boldsymbol{b}^{-2}$.

Then one needs to combine the expression for the one-loop contribution to the form factor from eqs. (16), (36), (37):

$$F^{qqV}_{LO}(Q^2;\xi) = W_0 + 2W^{(1)}_{LO}\left(\alpha_s, \frac{\mu^2}{\lambda^2}\right) + W^{(12)}_{LO}\left(\alpha_s, \frac{\mu^2}{\lambda^2}, \chi\right) + O(\alpha_s^2). \quad (38)$$

The high-energy asymptotic behavior of the form factor is determined by the (gauge-dependent) cusp anomalous dimension which stems from the renormalization of the Wilson integral (11) [11,12,14]:

$$\left(\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s} + \delta(\alpha_s,\xi)\xi\frac{\partial}{\partial\xi}\right)\ln F^{NS}(Q^2) = -\frac{1}{2}\Gamma^{qqg}_{\rm cusp}\left[\alpha_s(\mu^2);\chi\right], \quad (39)$$

with the one-loop functions

$$\beta(\alpha_s) = \mu^2\frac{\partial\alpha_s}{\partial\mu^2} = -\frac{\beta_0}{4\pi}\alpha_s^2 + O(\alpha_s^3), \quad \beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f, \quad (40)$$

and (see ref. [23])

$$\delta(\alpha_s;\xi) = \mu^2\frac{\partial\ln\xi}{\partial\mu^2} = \frac{\alpha_s}{4\pi}C_A\frac{\xi-1}{\xi}\left[\frac{\xi-1}{2} + \frac{13}{6}\right] + O(\alpha_s^2). \quad (41)$$

Thus, the calculation of the cusp anomalous dimension (39) yields in one-loop approximation

$$\Gamma^{qqg}_{\rm cusp}\left[\alpha_s(\mu^2);\chi;\xi\right] = \frac{\alpha_s}{\pi}\left[\left(C_F - \frac{C_A}{2}\right)\chi\coth\chi + \frac{C_A}{4}\xi - C_F\right] + O(\alpha_s^2), \quad (42)$$

where the Casimir operator of $SU(N_c)$ in the adjoint representation is used: $C_A = N_c$. Note, that in ref. [12], the term $C_A\xi/4$ in eq. (42) had *minus* sign. However, our expression allows to reproduce the color singlet case by means of the replacement of the color factors: $(C_F - C_A/2) \to C_F$, thus obtainig the gauge invariant result (2):

$$\Gamma^{qq\gamma}_{\rm cusp}\left[\alpha_s(\mu);\chi\right] = \frac{\alpha_s}{\pi}\left[C_F\chi\coth\chi + (C_F - C_F)\,\xi - C_F\right] + O(\alpha_s^2) = \frac{\alpha_s}{\pi}\frac{N_c^2-1}{2N_c}\left(\chi\coth\chi - 1\right) + O(\alpha_s^2), \quad (43)$$

yielding the well known Sudakov suppression. The latter can be considered as a convenient test of the calculations and confirms the accuracy of our result, eq. (42).

In order to find the large-Q^2 asymptotic, we take into account the limit of the large scattering angle:

$$\chi\coth\chi \propto \ln\frac{Q^2}{m^2}, \quad \chi \longrightarrow \infty, \quad (44)$$

and find that, in this limit, the anomalous dimension is linear in $\ln Q^2$ while the gauge-dependent term can be neglected:

$$\Gamma^{qqg}_{\rm cusp}\left[\alpha_s;\chi\right] = \Gamma^{qqg}_{\rm cusp}\left[\alpha_s\right]\ln\frac{Q^2}{m^2} + O(\ln^0 Q^2), \quad (45)$$

$$\Gamma^{qqg}_{\rm cusp}\left[\alpha_s\right] = \frac{\alpha_s}{\pi}\left(C_F - \frac{C_A}{2}\right) + O(\alpha_s^2) < 0. \quad (46)$$

Note that for the color group $SU(3)$, this anomalous dimension (46) has opposite sign compared to the color singlet case eq. (2): this is a direct consequence of the algebra of gauge group generators. Therefore, the leading (double-logarithmic) behavior of the non-singlet form factor is given by

$$F^{qqg}\left[Q^2\right]/F^{qqg}\left[Q_0^2\right] = \exp\left[-\left(C_F - \frac{C_A}{2}\right)\int_{Q_0^2}^{Q^2}\frac{{\rm d}\mu}{2\mu}\ln\frac{Q^2}{\mu}\frac{\alpha_s(\mu)}{\pi} + NLO\right] = \exp\left[\frac{2}{\beta_0 N_c}\ln\frac{Q^2}{\Lambda^2_{QCD}}\ln\frac{\ln Q^2/\Lambda^2_{QCD}}{\ln Q_0^2/\Lambda^2_{QCD}} + NLO\right], \quad (47)$$

that is the increasing function of Q^2. The dependence from the gauge-fixing parameter ξ drops out of the leading logarithmic expression for $\Gamma^{qqg}_{\rm cusp}$, eq. (46), and yields no influence on the main asymptotics.

5 Discussion and conclusions

We derived the general formula for the vacuum averaged Wilson integral (11) in the g^2 accuracy. This result eqs. (32), (33) is found in n dimensions, in an arbitrary covariant gauge, and is valid as well for any $SU(N_c)$ gauge field. In the case of a non-perturbative field $A^\mu_{\rm NP} \sim g^{-1}$, this result corresponds to the leading (so-called "weak field") order of expansion in field strength, while the dependence on the coupling g drops out.

By using the Wilson integrals techniques, it has been found that the cusp anomalous dimension for the non-singlet quark form factor eq. (46) has a negative sign —in contrast to the singlet case— what leads, in the large-Q^2 regime, to the enhancement, rather than suppression, of the contribution due to the resumed soft-gluon radiative corrections, eq. (47). The dependence on the gauge-fixing parameter ξ is shown to be of the order of $\ln^0 Q^2$ —not surviving in the asymptotical regime.

It is necessary to emphasize that we work here in the covariant gauge, whereas the case of the axial gauge is quite important and requires additional analysis (see, *e.g.*, refs. [24]). Another important version of the problem —the case of the *off*-shell quarks with different masses (applicable, *e.g.*, to flavor-changing processes)— is more technically involved and will be reported in the forthcoming work.

Another point which should be noted is that the quark-gluon vertex is a colored gauge-dependent object and could not be an observable quantity. In computations of the realistic processes when this vertex is inserted into the diagrams at the partonic level, the role of this enhancement can be reduced due to the emission of gluons, as well as due to the convolutions of the color indices.

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